

PROBABILITY ANALYSIS FOR PREDICTION OF ANNUAL MAXIMUM DAILY RAINFALL OF PHULBANI BLOCK OF KANDHAMAL DISTRICT ODISHA

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ABSTRACT

Kandhamla district is drought prone and hilly area so rainfall analysis will require finding out crop planning will be made accordingly to mitigate drought spell and drought situation for benefit of poor tribal farmers of the district. Probability analysis of annual maximum daily rainfall for different return periods has been suggested for the design of small and medium hydraulic structures. It has been observed that GEV distribution found suitable due to lowest Ch—square and lowest RMSE value. At 30% probability level the daily maximum rainfall of Phulbai can be taken as 160.3 mm, this can be taken for planning of different structures.

KEYWORDS: Probability Analysis, Crop Planning

INTRODUCTION

The light textured and well-drained upland soils classified under sandy loam (0-30 cm depth) in North Eastern Ghat Zone provide scope for cultivation of vegetables during rainy season. The intermittent dry spells and terminal drought affect the performance of these high value crops in most of the years. 36 to 50 % of the rainfall which is lost in the form of run-off. Harvesting of this run-off water in constructed farm tank with proper lining and reuse of this water for life-saving irrigation will protect the crop from dry spell occurred during *khariif*. If possible the harvested runoff water will be helpful for supplemental irrigation to a second crop after harvest of a first crop. Soil structure and organic matter status decide the water holding capacity of the soil. Soil physic-chemical characteristics depend on the systems of nutrient management. Keeping these points in view, the present experiment involving two water management systems i.e. pond and no pond based system have been designed.

Fragmented and small land holdings of small and marginal farmers restrict for construction of water harvesting structures / ponds in their lands. Further to mitigate the drought and to earn more profit Government implemented the 2nd green revolution and crop diversification programme in the uplands. Small farmers have affinity to grow paddy in uplands, because rice is their staple food. But the yield is very less due to drought and irregular rainfall pattern which make the farmer less profit, so it is necessary for these type of farmers to select the cropping systems, so that it will retain more moisture and simultaneously will conserve soil, this is no pond system in-situ management. This will made possible to grow a second crop in that area by utilizing the stored water in the pond, which is inferred as pond based cropping system. The medium and big farmers who can afford for construction of water harvesting pond in upland situations, high value remunerative crops with ex-situ management. This will also make to grow a second crop in that area, by utilizing the stored water in the pond. Hence for these type farmers, ex-situ management –pond based cropping system have been proposed.

So following objective has been considered:

To find out the one probability analysis of one day maximum rain fall data.

MATERIALS AND METHODS

Probability analysis of annual maximum daily rainfall for different return periods has been suggested for the design of small and medium hydraulic structures (Bhatt et.al. 1996). The probability analysis for prediction of annual maximum daily rainfall of Peryar basin in Kerala (George et.al. 2002), the incoming rainfall of different magnitudes and return periods can be predicted based on the theoretical probability distributions. This study is an attempt to find out the best fit of the observed annual maximum daily rainfall with the commonly used theoretical frequency distributions.

This chapter deals with review of some of the works done in the past, by the researchers on frequency analysis of hydrological events, forecasting models of hydrological events and crop-weather models etc.

Probability distributions are widely used for understanding the rainfall pattern and computation of minimum assured rainfall. A number of studies have been conducted to understand the process, which generates rainfall data in a given region.

Probability Distribution Functions

A lot of researches have been done to fit a distribution to precipitation data.

Normal Distribution

This is symmetrical, continuous distribution, theoretically representing the distribution of accidental errors about their mean, or the so called *Gaussian law of errors*. The probability density is

$$p(x) = (1/\sigma\sqrt{2\pi}) e^{-(x-\mu)^2/2\sigma^2}$$

Where x is the variate, μ is the mean value of variate and σ is the standard deviation. In this distribution, the mean, mode and median are the same. The cumulative probability of a value being equal to or less than x is

$$p(x \leq) = 1/\sigma\sqrt{2\pi} \int_{-\infty}^x e^{-(x-\mu)^2/2\sigma^2} dx$$

This represents the area under the curve between the variates of $-\infty$ and x .

Lognormal (2-Parameter) Distribution

This is transformed normal distribution in which the variate is replaced by its logarithmic value. This distribution represents the so called *law of Galton* as it was first studied by *Galton* in 1875. The probability density is

$$p(x) = (1/\sigma_y e^x \sqrt{2\pi}) e^{-(x-\mu_y)^2/2\sigma_y^2}$$

Where $y = \ln x$, where x is the variate, μ_y is the mean of y and σ_y is the standard deviation of y .

Log-Normal (3-Parameter) distribution

A random variable X is said to have three parameter log-normal probability distribution if its probability density function (pdf) is given by:

$$f(x) = \begin{cases} \frac{1}{(x-\lambda)\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\log(x-\lambda)-\mu}{\sigma}\right)^2\right\}, \lambda < x < \infty, \mu > 0, \sigma > 0 \\ 0, \text{otherwise} \end{cases}$$

Where μ , σ and λ are known as location, scale and threshold parameters, respectively.

Log-logistic distribution

The pdf of three parameter log-logistic probability distribution is given by:

$$f(x) = \begin{cases} \frac{e^{\{\log(x-\lambda)-\frac{\mu}{\sigma}\}}}{\sigma \left[1 + e^{\{\log(x-\lambda)-\frac{\mu}{\sigma}\}}\right]^2} & x > \lambda, \mu > 0, \sigma > 0 \\ 0, \text{otherwise} \end{cases}$$

Where μ , σ and λ are known as location, scale and threshold parameters, respectively.

Gamma Distribution

Probability density of this distribution is

$$p(x) = x^a e^{-x/b} / b(a+1)\Gamma(a+1)$$

With $b > 0$, $a > -1$ for $x > 0$

And $p(x) = 0$ for $x \leq 0$

Where a & b are constants and $\Gamma(a+1) = a!$ is a gamma function. The cumulative probability being equal to or less than $x (< \infty)$ is known as incomplete gamma function.

The statistical parameters are Mean = $b(a+1)$

And variance = $b^2(a+1)$

Pearson Distribution

Karl Pearson has derived a series of probability function to fit virtually any distribution. The general and basic equation to define the probability density of a Pearson distribution

$$p(x) = e \int_{-\infty}^x \frac{a+x}{b_0 + b_1x + b_2x^2} dx$$

Where a , b_0 , b_1 and b_2 are constants?

The criteria for determining types of distribution are β_1 , β_2 and k where

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$k = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$$

Where μ_2, μ_3 and μ_4 are second, third and fourth moments about the mean.

When $\beta_1 = 0, \beta_2 = 3$ and $k = 0$, the Pearson distribution is identical to the normal distribution. Chow (1964) suggested that Type-I and III distributions are often used in hydrologic frequency analysis.

2.1.6. (i) Type-I distribution:

For Type-I, $k < 0$. Its probability density is

$$p(x) = p_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}$$

Where $m_1/a_1 = m_2/a_2$ and the origin is at the mode.

The values of m_1 and m_2 are given by

$$m_1 \text{ or } m_2 = \frac{1}{2} \left[r - 2 \pm r(r+2) \frac{\sqrt{\mu_2 \beta_1}}{2(a_1 + a_2)} \right]$$

When μ_2 is positive, m_2 is the positive root and m_1 is the negative root and $r = \frac{6(\beta_2 - \beta_1 - 1)}{6 + 3\beta_1 - 2\beta_2}$

$$a_1 + a_2 = \frac{1}{2} \sqrt{\mu_2 [\beta_1 (r+2)^2 + 16(r+1)]}$$

$$\text{And } p_0 = \frac{N}{a_1 + a_2} \frac{m_1^{m_1} m_2^{m_2}}{(m_1 + m_2)^{(m_1 + m_2)}} \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1) \Gamma(m_2 + 1)}$$

Where N is the total frequency.

2.1.6. (ii) Type-III distribution:

For Type-III distribution

$$k = \infty \text{ or } 2\beta_2 = 3\beta_1 + 6$$

The probability density with the origin at mode is

$$p(x) = p_0 \left(1 + \frac{x}{a}\right)^c e^{-cx/a}$$

Where $c = \frac{4}{\beta_1} - 1$

$$a = \frac{c \mu_3}{2 \mu_2}$$

$$p_0 = \frac{Nc^{c+1}}{ae^c\Gamma(c+1)}$$

Log-Pearson Type Iii Distribution

This distribution is extensively used in USA for projects sponsored by the US Government. In this the variate is first transformed into logarithmic form (base 10) and the transformed data is then analyzed. If X is the variate of a random hydrologic series, then the series of Z variates where

$$z = \log x$$

are first obtained. For this z series, for any recurrence interval T and the coefficient of skew C_z ,

σ_z =Standard deviation of the Z variate sample

$$= \sqrt{\sum (z - \bar{z})^2 / (N - 1)}$$

And C_z =coefficient of skew of variate Z

$$= \frac{N \sum (z - \bar{z})^3}{(N-1)(N-2)\sigma_z^3}$$

\bar{z} =mean of z values

N = sample size = number of years of record

Generalized Pareto Distribution

The family of generalized Pareto distributions (GPD) has three parameters μ, σ and ξ .

The cumulative distribution function is

$$F_{(\xi, \mu, \sigma)}(x) = \begin{cases} 1 - \left(1 + \frac{\xi(x - \mu)}{\sigma}\right)^{-\frac{1}{\xi}} & \text{for } \xi \neq 0 \\ 1 - \exp\left(-\frac{x - \mu}{\sigma}\right) & \text{for } \xi = 0 \end{cases}$$

for $x \geq \mu$ when $\xi \geq 0$ and $x \leq \mu - \frac{\sigma}{\xi}$ when $\xi < 0$, where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$

the scale parameter and $\xi \in \mathbb{R}$ the shape parameter.

The probability density function is

$$f_{(\xi, \mu, \sigma)}(x) = \frac{1}{\sigma} \left(1 + \frac{\xi(x - \mu)}{\sigma}\right)^{\left(-\frac{1}{\xi}-1\right)}$$

Or

$$f_{(\xi, \mu, \sigma)}(x) = \frac{\sigma^{\frac{1}{\xi}}}{(\sigma + \xi(x - \mu))^{\left(\frac{1}{\xi}+1\right)}}$$

again, for $x \geq \mu$, and $x \leq \mu - \frac{\sigma}{\xi}$ when $\xi < 0$

Generalized Extreme Value Distribution

Generalized extreme value distribution has cumulative distribution function

$$F(x; \mu, \sigma, \xi) = \frac{1}{\sigma} \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{\left(\frac{-1}{\xi} - 1 \right)} \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{\frac{-1}{\xi}} \right\}$$

For $1 + \xi(x - \mu)/\sigma > 0$, where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ the scale parameter and $\xi \in \mathbb{R}$ the shape parameter. The density function is, consequently

$$f(x; \mu, \sigma, \xi) = \frac{1}{\sigma} \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{\left(\frac{-1}{\xi} - 1 \right)} \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{\frac{-1}{\xi}} \right\}$$

Again, for $1 + \xi(x - \mu)/\sigma > 0$

Gumbel's Method

The extreme value distribution was introduced by *Gumbel* (1941) and is commonly known as Gumbel's distribution. It is one of the most widely used probability-distribution functions for extreme values in hydrologic and meteorological studies. According to this theory of extreme events, the probability of occurrence of an event equal to or larger than a value x_0 is

$$P(X \geq x_0) = 1 - e^{-e^{-y}}$$

In which y is a dimensionless variable and is given by

$$y = \alpha(x - a)$$

$$a = \bar{x} - 0.45005\sigma_x$$

$$\text{Thus } y = \frac{1.2825(x - \bar{x})}{\sigma_x} + 0.577 \dots \dots \dots (i)$$

Where \bar{x} = mean and σ_x = standard deviation of the variate X . In practice it is the value of X for a given P that is required and such Eq. (i) is transposed as

$$y_p = -\ln[-\ln(1 - P)]$$

Noting that the return period $T = 1/P$ and designating y_T = the value of y , commonly called the reduced variate, for a given T

$$y_T = - \left[\ln \cdot \ln \frac{T}{T - 1} \right]$$

$$\text{Or } y_T = - \left[0.834 + 2.303 \log \log \frac{T}{T - 1} \right]$$

Now rearranging Eq. (i), the value of the variate X with a return period T is

$$x_T = \bar{x} + K\sigma_x$$

$$\text{where } K = \frac{(x_T - 0.577)}{1.2825}$$

The above equations constitute the basic Gumbel's equations and are applicable to an infinite sample size (i.e. $N \rightarrow \infty$).

Weibull Distribution

The Weibull distribution, also known as the Extreme Value Type III distribution, first appeared in his papers in 1939. The two-parameter version of this distribution has the density function

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right)$$

The Weibull distribution is defined for $x \geq 0$, and both distribution

Parameters (α -shape, β -scale) are positive. The two-parameter Weibull distribution can be generalized by adding the location (shift) parameter γ :

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x-\gamma}{\beta}\right)^\alpha\right)$$

In this model, the location parameter γ can take on any real value, and the distribution is defined for $x \geq \gamma$.

Goodness-of-fit Test

2.2.1. Chi-square Test:

Chi-square test of goodness of fit of observed values is calculated by following equation:

$$\chi_c^2 = \sum_{i=1}^K \frac{(O_i - E_i)^2}{E_i}$$

Where, K is the number of class interval, O_i and E_i are the observed and expected rainfall values in the i^{th} class, respectively. The distribution with least sum of χ_c^2 values will be adjudged the best. This distribution has been applied by *Subudhi* (2007) and *Senapati et. al.* (2009). Apart from Chi-square test other goodness-of-fit tests like Anderson-Darling test (AD) have been used by *Sharda and Das* (2005).

Plotting position

Subramanya (1990) has stated that the purpose of frequency analysis of an annual series is to obtain a relation between the magnitude of the event and its probability of exceedence. A simple empirical technique is to arrange the annual extreme series in descending order of magnitude the annual extreme series in descending order of magnitude and to assign an order number, m . The probability, p of an event equalled to or exceeded is given by Weibull formula

$$p = \frac{m}{N + 1}$$

Where m =rank number

N =number of years

The recurrence interval is given by

$$T = \frac{1}{p} = \frac{N + 1}{m}$$

Thom (1966) employed mixed gamma probability distribution for describing skewed rainfall data and employed approximate solution to non-linear equations obtained by differentiating log likelihood function with respect to the parameters of the distribution. Subsequently, this methodology along with variance ratio test (Cochran, 1954) as a goodness- of-fit has been widely employed (Mooley, 1973; Hargreaves, 1975; Sarkar et al., 1982; Biswas et al., 1989; Goel and Singh, 1999). Indian Meteorology Department (IMD) (1995) computed minimum assured amount of rainfall at 40, 50, 60 and 70% probability levels for different stations employing two-parameter gamma probability distribution. Stern and Coe (1984) and Hyndman and Grunwald (2000) employed gamma probability distribution for describing rainfall amount under generalised linear and additive model set up, respectively. Verma and Sarma (1988), Kar et. al (2004), Jat et. al (2006), Senapati et. al (2009) applied incomplete gamma probability distribution for rainfall analysis. In addition to gamma probability distribution, other two-parameter probability distributions (normal, log-normal, Weibull, smallest and largest extreme value), and three-parameter probability distributions (log-normal, gamma, log-logistic and Weibull) have been widely used for studying flood frequency (Kroll and Vogel, 2002; Ashkar and Mahdi, 2003; Clarke, 2003), drought analysis (Guttman, 1999; Quiring and Papakryiakou, 2003) and rainfall probability analysis (Subudhi, 2007; Senapati et. al.2009).

Slade (1936) is the first to fit a continuous probability distribution to the annual precipitation data. He found that the logarithmic transformed data has fitted to the normal distribution satisfactorily. Whitecomb (1940) fitted a Pearson type III distribution curve to monthly precipitation data.

Gumbel (1941), Brooks and Corruthers (1953), Gumbel (1954), Hershfield and Kohlar (1960). Hershfield(1962), Benson(1962), Hall and David(1963), Shane and Lynn(1964), Chow (1964), Markovic(1965),Yejevich(1972), Sales and Benyeden(1978) have applied gamma distribution with two and three parameter, Pearson type-III, extreme value, binomial and Poisson distribution to hydrological data.

Daily rainfall data from 1975 to 2010 were obtained for Phulbania block of Kandhamal district of Odisha

The return periods for various annual maximum daily rainfalls were calculated using Weibull’s formula.

$$T = \frac{n+1}{m} \tag{1}$$

Where T is the return period in years, n is the total number of years of record and m is the rank of the observed rainfall value after arranging them in descending order of magnitude. The probability of exceedence is the reciprocal of T values,i.e., if a hydraulic event equal to or greater than x occurs once in T years, the probability is equal to $1/T$.

Chow (1964) has given a general equation for the hydraulic frequency analysis,

$$x = \bar{x} + \sigma_x K \tag{2}$$

$$\text{or } \frac{x}{\bar{x}} = 1 + C_v K \tag{2a}$$

Where x is the event of specified probability, \bar{x} is the mean of the series, σ_x is the standard deviation. $C_v = \sigma_x / \bar{x}$ is the coefficient of variance and K , a frequency factor defined by a specific distribution, is a function of the probability level of x .

In applying the general equation, the statistical parameters required in the proposed distribution are first computed from the random factor K , which is a function of recurrence interval and the type of probability distribution can be determined from K - T relationship for the proposed distribution.

All the distribution and their return periods and skew ness co-efficient are available in standard tabular form (Hann-94)

Chi-Square Test of Goodness of Fit

The Chi-square value χ_c^2 can be calculated as:

$$\chi_c^2 = \sum_{i=1} \frac{(O_i - E_i)^2}{E_i} \tag{3}$$

Where O and E are the observed and expected rainfall values, respectively. The distribution with the least sum of C values will be adjudged the best.

RESULTS AND DISCUSSIONS

Table 1: Daily Maximum Rainfall Data at Phulbani, Mm

Year	Maximum Rainfall Daily, Mm
1968	154.4
1969	176.2
1970	187.8
1971	153.5
1972	137.6
1973	195.3
1974	69.6
1975	67.2
1976	86.2
1977	11.62
1978	138.6
1979	166
1980	128
1981	96.2
1982	484
1983	78
1984	225
1985	161
1986	227

1987	85
1988	88
1989	175
1990	112
1991	259
1992	237
1993	143
1994	151
1995	68
1996	70
1997	143
1998	80.3
1999	107
2000	70
2001	117
2002	82
2003	182
2004	90
2005	214.2
2006	203.8
2007	116.4
2008	124
2009	128.4
2010	98

Table 1 shows the value of one day maximum rainfall in different years from 1968 to 2010.

Table 2: Probability Values in, Mm by GEV Distribution

Obs. No.	%of probability	Amount, mm
1	10	233.8
2	20	187.3
3	30	160.3
4	40	140.7
5	50	124.7
6	60	110.6
7	70	97.2
8	80	83.5
9	90	67.2

Table 2 shows the value of rainfall in different probability by GEV distribution which has lowest RMSE value, so best fitted distribution among all the distribution using Flood software.

Table.3 Chi-Square Values of Different Distribution

Distribution	Computed Value	Calculated Value
GEV	1.233	1.39
Log pearson	1.791	1.83
Gamma	2.628	3.67
Pearson	2.628	2.77
Log(3 parameter)	2.3	2.41
Lognormal	3.186	3.67
Gumbel(Extreme value min)	7.651	7.81

Table 3 shows that GEV distribution has got lowest computed chi-square value compared to calculated chi-square value.

The daily maximum value at 30 % (160.3mm) and 70% (97.2mm) probability may be used for design of soil conservation structures and crop planning of Kandhamal district of Odisha. Flood software was used to find out the suitable distribution.

CONCLUSIONS

It can be concluded that the 30% probability level the daily maximum rainfall of Phulbai can be taken as 160.3 mm, this can be taken for planning of different structures. The distribution GEV fitted best as it has got lowest Chi-square value compared to other distribution and lowest RMSE b value.

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